Deteriorating Manufacturing System with Selling Price Discount under Random Machine Breakdown

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Abstract

This paper extends an imperfect production inventory model to machine breakdown at a random time and rework of defective items. Any manufacturing system wants to produce perfect quality items. But in long run production process, there may arise different types of difficulties like labor problem, rawmaterial problems, operator problems, etc., due to that the machinery systems shift from in-control state to out-of-control state as a result the manufacturing systems produce mixture of perfect and imperfect quality items. Some portion of imperfect items are reworked at a cost to become the perfect one. The development cost may be increased the in-control state and reduced the imperfect production. We want to maximize the expected total profit is illustrated by numerical examples and also its sensitivity analysis is carried out.

Keywords: Imperfect production, Inventory, Machine breakdown, Development cost.

1. Introduction

Many researchers have initiated to analyze various problems related to economic production quantity (EPQ) model (cf. Silver and Peterson [24], Rosenblatt and Lee [20], Hariga and Ben-Daya [14], Salameh and Jaber [21], Cárdenas-Barrón [6], Sana [22], Manna et al. [18] and others). Some of them, ignore various assumptions in the classical economic production quantity (EPQ) model. One of them that the production system is free of failures and that all items produced are perfect (cf. Silver and Peterson [24]). In a real manufacturing system, it is seen that initially in the production cycle, the production process is in an in-control state, because every factors associated with the system are fresh. But, due to continuous running of system these factors gradually losses their perfectness. Sana [22] have developed an economic production lot size model in an imperfect production system where the production process shifts from the ‘in-control’ state to an ‘out-of-control’ state after a random time and during ‘out-of-control’ state, imperfect quality items are produced. Pal et al. [19] has presented a mathematical model on EPQ for stochastic demand in an imperfect production system in which ‘out-of-control’ state, the system produces more defective items than in the ‘in-control’ state.

In real-life production systems, the rework option plays an important role in eliminating waste and affecting the cost of manufacturing. items. Many researchers shown that reworking of defective items and process minimal repair lead to raising the expected profit. Zhang and Gerchak [25] have considered a joint lotsizing and inspection policy studied under an EOQ model where a random proportion of units are defective. These defectives cannot be used and thus must be replaced by non-defective ones. Goyal and Cardenas-Barron [13] have reworked on the paper of Salameh and Jaber [21] and presented practical approach to find out the optimal lotsize. Hayek and Salameh [15] and Chiu [4] discussed the optimal production lot size model with reworking of defective items. Flappera and Teunterb [12] studied how reworking plans could both reduce the system costs and be environmental friendly. Inderfurth et al. [16] analyzed a cost minimizing-scheduling of work and rework processes on a single facility under deterioration of reworkable items. Chen [2] and Chiu et al. [4] reviewed a more general model that allowed a certain proportion of reworked units also may be scrapped. Sheu and Chen’s [23] and Chiu et al. [4] extensions on Ben-Daya [1] to incorporate minimal repair or rework considerations. Recently, Manna et al. [17] developed an EPQ model with promotional demand in random planning horizon in which some portion of defective items are reworked at a cost.

In the proposed model, we developed economic production quantity (EMQ) model for imperfect items and also we considered machine break-down at a random time. we consider during production-run-time, the production process may shift from in-control-state to an 'out-of-control' state after a random time that follows exponential distribution function. We assumed that the defective rate in in-control state is less than the defective rate in out-control state. We have been formulated a profit function, and maximized it by considering the production time and production rate as decision variables. The paper is organized as follows: Section 2 presents fundamental assumptions and notation. Section 3 mathematical formulation of the model. Section 4 provides numerical examples. Sensitivity analysis is discussed in Section 5. Section 6 concludes the paper.

2. Assumptions And Notations

The following assumptions and notations are used in developing the model.

2.1 Notations:

For convenience, the following notations are used throughout the entire paper.

p : The production rate is constant and deterministic.

D : Demand rate of items.

τ : Random time with mean $1/\lambda$ after which the system shifts from an "in-control" state to an "out-of-control" state for manufacturer.

$f(\tau)$: Probability density function of τ.

t_b : Random time when machine break down occur.

$g(t_b)$: Probability density function of t_b.

θ_1 : Percentage of defective items produced in in-control state.

θ_2 : Percentage of defective items produced in out-of-control state ($\theta_1 < \theta_2$).

δ : Probability of rework rate of defective units per unit time.

c_p : Production cost per unit.

c_s : Screening cost per unit item.

c_h : The inventory holding cost per unit time for products in production center.

K : Setup cost per cycle of production system.

c_r : Average reworking cost per unit item for manufacturer.

c_v : Devolvement cost.

A : $=(A_0 + K/P)$, set up cost of retailer.

s : Selling price per unit item sold for perfect quality.

t_b : Production period.

T : Total business period.

2.2 Assumptions :

The mathematical model of the proposed inventory problem is based on the following assumptions:

(i) In a production system, a manufacturer produces a mixture of defective and non-defective quality items and some portion of defective items are reworked at a cost.

(ii) In the production system, after a time (τ), the production process may shift from the in-control state to out-control state. The time (τ) is an exponential distributed with a finite mean.

(iii) According to assumption (ii), every one connected to the production has capability to give its perfection to produce an item perfectly. So, during the period $(0,\tau)$, obviously the number of imperfect items is small in amount but after this period it will be high. Therefore, it is assumed that the defective rate (θ_1) in in-control state is less than the defective rate (θ_2) in out-control state and is given by

$$\theta = \begin{cases} \theta_1, & 0 \le t \le \tau \\ \theta_2, & \tau \le t \le t_p \end{cases} \quad (1)$$

where θ_1 and θ_2 ($\theta_1 < \theta_2$) be the percentages of defective items to be produced in in-control state and out-of-control state respectively. Here the defective rates θ_1 and θ_2 are considered to be distributed uniformly with a finite mean and variance.

(iv) Due to long run production process and increasing the duration of in-control state, refine production methods and reduce production costs we consider devolvement cost (c_v)as the form $c_v = f(\tau)$,

$$f(\tau)=\begin{cases} B_0, & 0\le t\le\tau \\ B_0+B_1(t-\tau)e^{k_1\frac{\upsilon_{max}-\upsilon}{\upsilon-\upsilon_{min}}}, & \tau\le t\le t_p \end{cases} \quad (2)$$

(v) In this model, we assume that the demand rate depend on selling price discount. The demand rate is defined as

$$D = D_0 - \rho e^{ks} \quad (3)$$

where ρ and k is the effective parameter of demand rate on discount.

(vi) Production period (t_p) is decision variable.

(vii) The time horizon is infinite.

3. Mathematical formulation of proposed inventory model and discussion:

This model considers a supply chain system between manufacturer and customer for single type of products such as mobiles, in which the qualities of the production process and inspection process are not perfect. In this manufacturing system, it is considered that production, inspections and reworked processes are performed simultaneously. Here production is started at a rate of P from the beginning and it continues upto the end of the production run, t_p. During the whole production period all produced items are inspected at the rate of P. Initially, the production system starts from in-control state and continues to any random time, τ from which in-control state shifts to out-of-control state and it stays in out-of-control state until the end of the production-run, t_p. According to assumption (iv), the probability of the number of defective items in in-control state is less than the probability of the number of defective items in out-of-control state. The consumption process continues at the customer demand rate (D) until the end of business cycle time T. Since the time t_b is random time for which the machine break down occur, depending on the position of t_b, the model has different cases which are discussed as follows:

3.1. Case 1: when the "out-of-control" state to be occurred during the production-run time, i.e, $0<\tau<t_p$.

In this case, the production period $[0,t_p]$ can be divided into two sub-intervals such as $[0,\tau]$ and $[\tau,t_p]$. During the time interval $[0,\tau]$, the production process is in in-control state and in $[\tau,t_p]$ the process is in out-of-control state. Through-out the time interval $[0,\tau]$, the amount of non-defective items, defective items and reworked items are $(1-\theta_1)P\tau$, $\theta_1 P\tau$ and $\delta\theta_1 P\tau$ respectively. Also on $[\tau,t_p]$, the amount of non-defective items, defective items and reworked items are $(1-\theta_2)P(t_p-\tau)$, $\theta_2 P(t_p-\tau)$ and $\delta\theta_2 P(t_p-\tau)$ respectively.

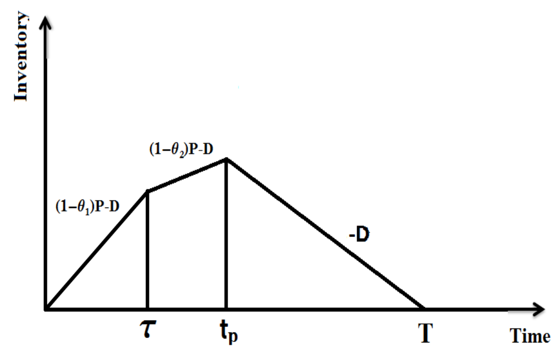

Figure 1: Graphical representation of inventory of selling item.

During the period $[0,t_p]$, the inventory level increases due to excess production after fulfill the customer demand upto time $t=t_p$ at which the inventory level reaches at maximum. Therefor the behavior of the inventory level during the interval $[0,\tau]$ and $[\tau,t_p]$ are given by

$$I_1(t)=[(1-\theta_1+\delta\theta_1)P-D]t, \qquad 0\le t\le\tau$$

and $I_2(t)=[(1-\theta_2+\delta\theta_2)P-D](t-\tau)+I_1(\tau), \tau\le t\le t_p$

$$=[(1-\theta_2+\delta\theta_2)P-D]t+(1-\delta)(\theta_2-\theta_1)P\tau, \tau\le t\le t_p$$

Then during the period $[t_p,T]$ the inventory level decline due to meeting customer demand and it reaches zero at T. Therefor the behavior of the inventory level during the interval $[t_p,T]$ is given by

$I_3(t) = D(T-t), t_p \le t \le T$

Lemma 1: When $0 < \tau \le t_p$, in a manufacturing system the business period (T) must satisfy the following relation in terms of production rate (P), demand rate (D) and production period (t_p)

$$T_1(\tau, t_p) = \frac{1}{D}[(1-\theta_2+\delta\theta_2)Pt_b + (1-\delta)(\theta_2-\theta_1)P\tau]$$

Proof: Satisfying the continuity condition of $I_2(t)$ and $I_3(t)$ at $t = t_p$ a relation is obtain following

$$[(1-\theta_2+\delta\theta_2)P-D]t_p + (1-\delta)(\theta_2-\theta_1)P\tau = D(T-t_p)$$

$$i.e, T = \frac{1}{D}[(1-\theta_2+\delta\theta_2)Pt_p + (1-\delta)(\theta_2-\theta_1)P\tau]$$
$$= T_{11}(\tau, t_p), say$$

Now, the proof is complete.

Holding cost:

During the period $[0,T]$, the holding cost is given by

$$HC_1(\tau,t_p) = h_c[\int_0^{\tau} I_1(t)dt + \int_{\tau}^{t_p} I_2(t)dt + \int_{t_p}^{T} I_3(t)dt]$$
$$= \frac{h_c}{2}[\{(1-\theta_1+\delta\theta_1)P-D\}\tau^2 + \{(1-\theta_2+\delta\theta_2)P-D\}(t_p^2-\tau^2)$$
$$+(1-\delta)(\theta_2-\theta_1)P\tau(t_p-\tau) + D(T-t_p)^2]$$
$$= \frac{h_c}{2}[\{(1-\theta_1+\delta\theta_1)P-D\}\tau^2 + \{(1-\theta_2+\delta\theta_2)P-D\}(t_p^2-\tau^2)$$
$$+(1-\delta)(\theta_2-\theta_1)P\tau(t_p-\tau) + \frac{1}{D}\{\{(1-\theta_2+\delta\theta_2)P-D\}^2 t_p^2$$
$$+(1-\delta)^2(\theta_2-\theta_1)^2P^2\tau^2 + 2\{(1-\theta_2+\delta\theta_2)P-D\}$$
$$(1-\delta)(\theta_2-\theta_1)Pt_b\tau\}]$$
$$= \frac{h_c}{2}[(1-\delta)(\theta_2-\theta_1)P\tau t_p + \{(1-\theta_2+\delta\theta_2)P-D\}t_p^2$$
$$+\frac{1}{D}\{\{(1-\theta_2+\delta\theta_2)P-D\}^2 t_p^2 + (1-\delta)^2(\theta_2-\theta_1)^2P^2\tau^2$$
$$+2\{(1-\theta_2+\delta\theta_2)P-D\}(1-\delta)(\theta_2-\theta_1)Pt_b\tau\}]$$

Manufacturing, inspection and reworked cost:

During the period $[0,t_p]$, total manufacturing, inspection and reworked cost is given by

$$PC_1(\tau,t_p) = [(c_p+c_s)Pt_b + c_r\delta\{\theta_1P\tau + \theta_2P(t_p-\tau)\}]$$
$$= (c_p+c_s+c_r\delta\theta_2)Pt_b + c_r\delta(\theta_1-\theta_2)P\tau$$

Development cost:

During the period $[0,t_p]$, the development cost is given by

$$DVC_1(\tau,t_p) = [\int_0^{\tau} B_0dt + \int_{\tau}^{t_p}\{B_0 + B_1(t-\tau)e^{k\frac{\upsilon_{max}-\upsilon}{\upsilon-\upsilon_{min}}}\}dt]$$
$$= B_0t_p + \frac{B_1}{2}(t_p-\tau)^2 e^{k_1\frac{\upsilon_{max}-\upsilon}{\upsilon-\upsilon_{min}}}$$

Revenue from serviceable items:

During the period $[0,T]$, the amount of serviceable items is $(1-\theta_1+\delta\theta_1)Pt_b$ Thus, the total sales revenues during the interval $[0,T]$ is given by

$$SR_1(\tau,t_p) = s\{(1-\theta_1+\delta\theta_1)P\tau + (1-\theta_2+\delta\theta_2)P(t_p-\tau)\}$$
$$= s\{(1-\delta)(\theta_2-\theta_1)P\tau + (1-\theta_2+\delta\theta_2)Pt_b\}$$

Therefore, the profit function during $0<\tau<t_p$ of production system is given by

$$\Pi_1(\tau,t_p) = s\{(1-\delta)(\theta_2-\theta_1)P\tau + (1-\theta_2+\delta\theta_2)Pt_p\}$$
$$-\{(c_p+c_s+c_r\delta\theta_2)Pt_p + c_r\delta(\theta_1-\theta_2)P\tau\}$$
$$-\frac{h_c}{2}[(1-\delta)(\theta_2-\theta_1)P\tau t_p + \{(1-\theta_2+\delta\theta_2)P-D\}t_p^2$$
$$+\frac{1}{D}\{\{(1-\theta_2+\delta\theta_2)P-D\}^2t_p^2 + (1-\delta)^2(\theta_2-\theta_1)^2P^2\tau^2$$
$$+2\{(1-\theta_2+\delta\theta_2)P-D\}(1-\delta)(\theta_2-\theta_1)Pt_p\tau\}] - (A_0 + \frac{K}{P})$$
$$-B_0t_p - \frac{B_1}{2}(t_p-\tau)^2 e^{k_1\frac{\upsilon_{max}-\upsilon}{\upsilon-\upsilon_{min}}}$$

3.2. Case 2: when the "out-of-control" state not to be occurred during the production-run time, i.e, $0 \le t_p \le \tau$.

In this case, the production process goes to out-of-control state after machine break down. But production process stopped due to machine break down. So, during the production period $[0,t_p]$ the production process is in in-control state. Through-out the time interval $[0,t_p]$, the amount of non-defective items, defective items and reworked items are $(1-\theta_1)Pt_b$, θ_1Pt_b and $\delta\theta_1Pt_b$ respectively.

During the period $[0, t_p]$, the inventory level increases due to excess production after fulfill the customer demand upto time $t = t_p$ at which the inventory level reaches at maximum. Therefor the behavior of the inventory level during the interval $[0, t_p]$ is given by

$$I_1(t) = [(1-\theta_1+\delta\theta_1)P - D]t, 0 \le t \le t_p$$

Then during the period $[t_p, T]$ the inventory level decline due to meeting customer demand and it reaches zero at T. Therefor the behavior of the inventory level during the interval $[t_p, T]$ is given by $I_2(t) = D(T-t), t_p \le t \le T$

Lemma 2: When $0 \le t_p \le \tau$, in a manufacturing system the business period (T) must satisfy the following relation in terms of production rate (P), demand rate (D) and production period (t_p), is, $T_2(t_p) = \frac{1}{D}(1-\theta_1+\delta\theta_1)Pt_b$

Proof: Satisfying the continuity condition of $I_1(t)$ and $I_2(t)$ at $t = t_p$ a relation is obtain following

$$[(1-\theta_1+\delta\theta_1)P - D]t_p = D(T - t_p)$$

$$i.e, T = \frac{1}{D}(1-\theta_1+\delta\theta_1)Pt_b = T_{12}(t_p), say$$

Now, the proof is complete.

Holding cost:

During the period $[0, T]$, the holding cost is given by $HC_2(t_p) = h_c[\int_0^{t_p} I_1(t)dt + \int_{t_p}^{T} I_2(t)dt]$

$$= \frac{h_c}{2}[\{(1-\theta_1+\delta\theta_1)P - D\}t_p^2 + \frac{1}{D}\{(1-\theta_1+\delta\theta_1)P - D\}^2 t_p^2]$$

Manufacturing, inspection and reworked cost:

During the period $[0, t_p]$, total manufacturing, inspection and reworked cost is given by

$$PC_2 = (c_p + c_s + c_r\delta\theta_1)Pt_b$$

Development cost:

During the period $[0, t_p]$, the development cost is given by

$$DVC_2 = \int_0^{t_p} B_0 dt = B_0 t_p$$

Revenue from serviceable items:

During the period $[0, T]$, the amount of serviceable items is $(1-\theta_1+\delta\theta_1)Pt_b$ Thus, the total sales revenues during the interval $[0, T]$ is given by

$$SR_2 = s(1-\theta_1+\delta\theta_1)Pt_b$$

Therefore, the profit function during $0 < t_p < \tau$ of production system is given by

$$\Pi_2(t_p) = s(1-\theta_1+\delta\theta_1)Pt_b - (c_p + c_s + c_r\delta\theta_1)Pt_b - B_0 t_p$$
$$-\frac{h_c}{2}[\{(1-\theta_1+\delta\theta_1)P - D\}t_p^2 + \frac{1}{D}\{(1-\theta_1+\delta\theta_1)P - D\}^2 t_p^2]$$
$$-(A_0 + \frac{K}{P})$$

The production process shifted or not shifted from "in-control" state to "out-of-control" state the expected profit function of the production system w.r.t τ is given by

$$\Pi(t_p) = \int_0^{t_p} \Pi_1(t_p, \tau) f(\tau) d\tau + \int_{t_p}^{\infty} \Pi_2(t_p) f(\tau) d\tau$$
$$= s\{(1-\theta_2+\delta\theta_2)Pt_p + \frac{1}{\lambda}(1-\delta)(\theta_2-\theta_1)P(1-e^{-\lambda t_p})\}$$
$$-(c_p + c_s + c_r\delta\theta_2)Pt_p - \frac{c_r}{\lambda}\delta(\theta_1-\theta_2)P(1-e^{-\lambda t_p}) - (A_0 +$$
$$\frac{K}{P}) - B_0 t_p - \frac{B_1}{2}\{t_p^2 - \frac{2t_p}{\lambda} + \frac{2}{\lambda^2}(1-e^{-\lambda t_p})\}e^{k_1\frac{\upsilon_{max}-\upsilon}{\upsilon-\upsilon_{min}}}$$

$$-\frac{h_c}{2}[\{(1-\theta_2+\delta\theta_2)P - D\}t_p^2 + \frac{1}{\lambda}(1-\delta)(\theta_2-\theta_1)Pt_p$$
$$(1-e^{-\lambda t_p}) + \frac{1}{D}\{\{(1-\theta_2+\delta\theta_2)P - D\}^2 t_p^2$$
$$+(1-\delta)^2(\theta_2-\theta_1)^2 P^2\{\frac{2}{\lambda^2} - \frac{1}{\lambda^2}(2 + 2\lambda t_p + \lambda^2 t_p^2)e^{-\lambda t_p}\}$$
$$+2\{(1-\theta_2+\delta\theta_2)P - D\}(1-\delta)(\theta_2-\theta_1)Pt_p$$
$$\{\frac{1}{\lambda}(1-e^{-\lambda t_p}) - t_p e^{-\lambda t_p}\}\}]$$

The production process shifted or not shifted from "in-control" state to "out-of-control" state the expected business period of the production system w.r.t τ is given by

$$T(t_p)=\int_0^{t_p}T_1(t_p,\tau)\,f(\tau)\,d\tau+\int_{t_p}^{\infty}T_2(t_p)\,f(\tau)\,d\tau$$

$$=\frac{1}{D}\int_0^{t_p}[(1-\theta_2+\delta\theta_2)Pt_p+(1-\delta)(\theta_2-\theta_1)P\tau]\lambda e^{-\lambda\tau}\,d\tau$$

$$+\frac{1}{D}(1-\theta_1+\delta\theta_1)Pt_p\int_{t_p}^{\infty}\lambda e^{-\lambda\tau}\,d\tau$$

$$=\frac{1}{D}[(1-\theta_2+\delta\theta_2)Pt_p+\frac{1}{\lambda}(1-\delta)(\theta_2-\theta_1)P(1-e^{-\lambda t_p})]$$

Now if the machine is break down at any random time $t=t_b$ with density function $g(t_b)=\mu e^{-\mu t_b}, t_b>0$. Then

$$\Pi=\begin{cases}\Pi(t_b), & if \quad t_b<t_p\\ \Pi(t_p), & if \quad t_b>t_p\end{cases} \quad (4)$$

and

$$T=\begin{cases}T(t_b), & if \quad t_b<t_p\\ T(t_p), & if \quad t_b>t_p\end{cases} \quad (5)$$

Therefore, with or with out machine break down in the production process, we have the expected profit function of the production system w.r.t t_b is given by

$$TP(t_p)=\int_0^{t_p}\Pi(t_b)\,g(t_b)\,dt_b+\int_{t_p}^{\infty}\Pi(t_p)\,g(t_b)\,dt_b$$

$$=s\{\frac{1}{\mu}(1-\theta_2+\delta\theta_2)P(1-e^{-\mu t_p})+\frac{1}{\mu+\lambda}(1-\delta)(\theta_2-\theta_1)$$

$$P\{1-e^{-(\lambda+\mu)t_p}\}\}-\frac{1}{\mu}(c_p+c_s+c_r\delta\theta_2)P(1-e^{-\mu t_p})$$

$$-\frac{c_r}{\mu+\lambda}\delta(\theta_1-\theta_2)P\{1-e^{-(\lambda+\mu)t_p}\}-(A_0+\frac{K}{P})-$$

$$\frac{B_0}{\mu}(1-e^{-\mu t_p})-\frac{B_1}{\mu}\{(\frac{1}{\mu}+\frac{1}{\lambda})(1-e^{-\mu t_p})-t_pe^{-\mu t_p}$$

$$+\frac{\mu}{\lambda(\lambda+\mu)}\{1-e^{-(\lambda+\mu)t_p}\}\}e^{k_1\frac{\upsilon_{max}-\upsilon}{\upsilon-\upsilon_{min}}}$$

$$-\frac{h_c}{2}[\frac{2}{\mu}\{(1-\theta_2+\delta\theta_2)P-D\}\{\frac{1}{\mu}(1-e^{-\mu t_p})-t_pe^{-\mu t_p}\}$$

$$+\frac{1}{\lambda}(1-\delta)(\theta_2-\theta_1)P\{\frac{1}{\mu}(1-e^{-\mu t_p})-\frac{\mu}{\lambda+\mu}\{1-e^{-(\lambda+\mu)t_p}\}\}$$

$$+\frac{2}{D\mu}\{\{(1-\theta_2+\delta\theta_2)P-D\}^2\{\frac{1}{\mu}(1-e^{-\mu t_p})-t_pe^{-\mu t_p}\}$$

$$+(1-\delta)^2(\theta_2-\theta_1)^2P^2\{\frac{2}{(\lambda+\mu)^2}(1-e^{-(\lambda+\mu)t_p})-$$

$$\frac{2}{\mu+\lambda}t_pe^{-(\mu+\lambda)t_p}\}\{1-\frac{2\mu}{\lambda(\mu+\lambda)}\}\}+2\{(1-\theta_2+\delta\theta_2)P-D\}$$

$$(1-\delta)(\theta_2-\theta_1)P\{\frac{1}{\lambda}\{\frac{1}{\mu}(1-e^{-\mu t_p})-\frac{\mu}{(\mu+\lambda)^2}\{1-e^{-(\mu+\lambda)t_p}\}$$

$$-\frac{\lambda}{\mu+\lambda}t_pe^{-(\mu+\lambda)t_p}\}-\frac{2\mu}{\lambda(\mu+\lambda)}\{\frac{2}{(\mu+\lambda)^2}(1-e^{-(\mu+\lambda)t_p})$$

$$-\frac{2}{(\mu+\lambda)}t_pe^{-(\mu+\lambda)t_p}\}\}] \quad (6)$$

Therefore, the expected business period of the production system w.r.t t_b is given by

$$T(t_p)=\int_0^{t_p}T(t_b)\,g(t_b)\,dt_b+\int_{t_p}^{\infty}T(t_p)\,g(t_b)\,dt_b$$

$$=\int_0^{t_p}\frac{1}{D}(1-\theta_2+\delta\theta_2)Pt_b\mu e^{-\mu t_b}\,dt_b$$

$$+\int_0^{t_p}\frac{1}{\lambda D}(1-\delta)(\theta_2-\theta_1)P(1-e^{-\lambda t_b})\mu e^{-\mu t_b}\,dt_b$$

$$+\int_{t_p}^{\infty}\frac{1}{D}(1-\theta_2+\delta\theta_2)Pt_p\mu e^{-\mu t_b}+$$

$$\int_{t_p}^{\infty}\frac{1}{\lambda D}(1-\delta)(\theta_2-\theta_1)P(1-e^{-\lambda t_p})\mu e^{-\mu t_b}\,dt_b$$

$$=\frac{P}{D}[\frac{1}{\mu}(1-\theta_2+\delta\theta_2)(1-e^{-\mu t_p})+\frac{1}{\lambda+\mu}(1-\delta)(\theta_2-\theta_1)$$

$$\{1-e^{-(\mu+\lambda)t_p}\}] \quad (7)$$

Average Profit of the production system during the period $[0,T]$ is given by

$$AP(t_p)=\frac{Expected \quad profit}{Expected \quad business \quad period}$$

$$=\frac{\int_0^{t_p}\Pi(t_b)\,g(t_b)\,dt_b+\int_{t_p}^{\infty}\Pi(t_p)\,g(t_b)\,dt_b}{\int_0^{t_p}T(t_b)\,g(t_b)\,dt_b+\int_{t_p}^{\infty}T(t_p)\,g(t_b)\,dt_b}=\frac{TP(t_p)}{T(t_p)} \quad (8)$$

4. Solution:

Our objective function(Expected profit per unit time) $AP(t_p)$ is a function of variable t_p ,when consider the selling price of the item is fixed.. So , our objective is to find the optimal value of t_p , for which the objective function $AP(t_p)$ is maximum.

So that the necessary condition for objective function to be maximised is $\frac{\partial AP(t_p)}{\partial t_p} = 0$ and the sufficient condition $\frac{\partial^2 AP(t_p)}{\partial t_p^2} < 0$ for $t_p > 0$. Now due to highly nonlinearity of $AP(t_p)$, the first and second derivatives of this with respect to t_p are very much complicated . Hence it is difficult to determine closed form solution of t_p for which the objective $AP(t_p)$ is optimum by analytical method. Using MATHEMATICA , we see that, Expected profit per unit time is concave for $t_p > 0$.

5. Numerical Examples:

The parametric values in the model are as K=1000,P=400, $D_0 = 400$, $\rho = 0.5$, $k = 0.05$, $s = 80$, $\delta = 0.2$, $\lambda = 0.15$, $\mu = 0.2$, $\theta_1 = 0.1$, $\theta_2 = 0.3$, $v_{max} = 50$, $v_{min} = 20$, $v = 40$, $c_h = 5$, $c_p = 20$, $c_r = 2$, $c_s = 5$, $A_0 = 500$, $B_0 = 100$, $B_1 = 10$, $k_1 = 0.2$,We analyses the effect of Expected profit per unit time on optimum production runtime of the production process

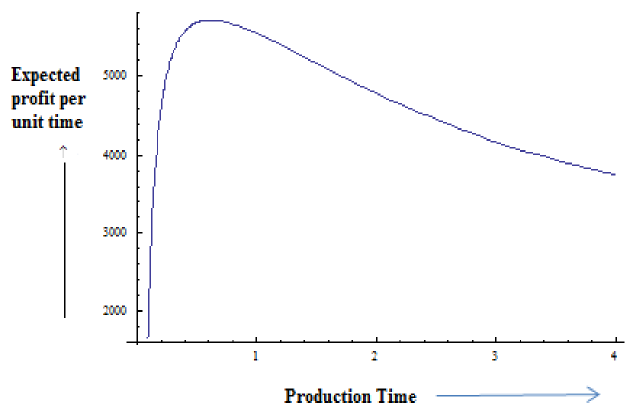

Figure 2: Expected Profit per unit time is concave with respect to the variable t_p .

Table-1: Expected Optimum profit per unit time , Optimum production time .

Optimum Production run time(t_p^*)	Optimum Expected Profit
0.611837	5712.59

Table 2: Expected optimum production time and expected optimum Profit/unit time with respect to different change of parameter.

Parameter	Change in Parameter	Optimum Production Time	Optimum Expected Profit/Unit Time
p	-10%	0.479205	5979.30
	-5%	0.531734	5833.63
	0%	0.611837	5712.39
	+5%	0.754778	5633.13
	+10%	1.146560	5545.31
s	-10%	0.538452	2944.52
	-5%	0.561068	4354.57
	0%	0.611837	5712.39
	+5%	0.700937	7011.87
	+10%	0.904269	8262.26
K	-10%	0.611668	5712.83
	-5%	0.611752	5712.61
	0%	0.611837	5712.39
	+5%	0.611921	5712.17
	+10%	0.612005	5711.95
θ_1	-10%	0.621700	4725.28
	-5%	0.616372	5223.34
	0%	0.611837	5712.39
	+5%	0.608044	6192.29
	+10%	0.604955	6662.89
θ_2	-10%	0.730175	9177.33
	-5%	0.665893	7507.55
	0%	0.611837	5712.39
	+5%	0.565713	3791.82
	+10%	0.525877	1745.79
v	-10%	0.611796	5700.67
	-5%	0.611819	5707.29
	0%	0.611837	5712.37
	+5%	0.611851	5716.44
	+10%	0.611862	5719.73

6.. Sensitivity Analysis :

Since the shifting time point from in–control state to out-of-control state is random variable in between beginning and end of the production run and Machine may breakdown at any random time point. so, the optimum

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production time and optimum profit per unit time is depend on that.

From Table-2 , Figure-3, it is observed that optimum production time is increases with increase of production rate ,rate selling price ,variable setup cost and variable development cost and it is decreases with the increase of other parameter ,rate of defective items in 'in-control' state and rate of defective items in 'out-of--control' state.

Similarly from Table-2 , Figure-4 ,it follows that optimum profit per unit time is increases with the increases of selling price rate, rate of defective items in 'in-control' state and variable development cost and decreases with respect to all other parameter.

Optimum production time is sensitive to the change of parameters production rate(p) and selling price rate(s) and optimum profit per unit time is sensitive to the change of parameter production rate(p). ,selling price rate(s).and is highly sensitive with the rate of defective items in 'out-of-control' state. It is insensitive with change of variable setup cost and variable development cost and moderately sensitive with all other parameters.

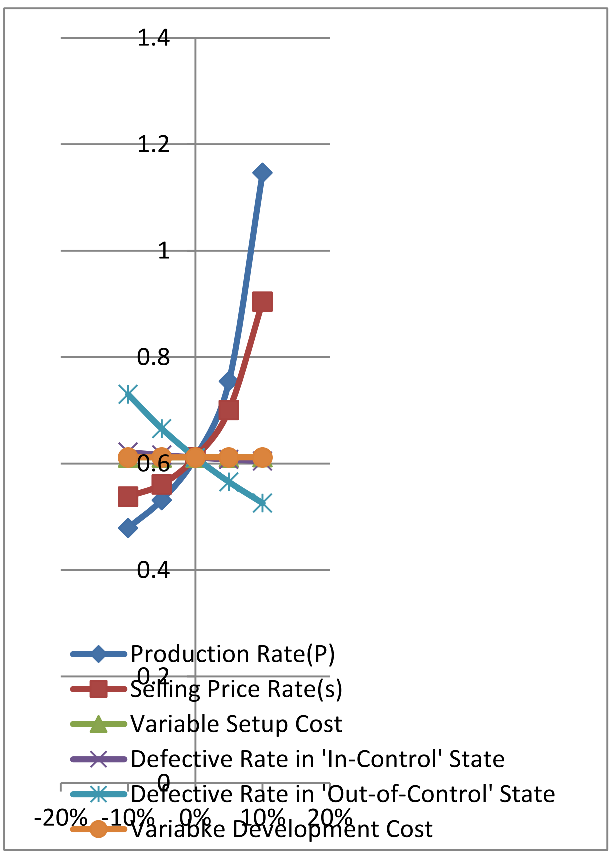

Figure 3: **Production time with respect to Percentage change of different parameter**

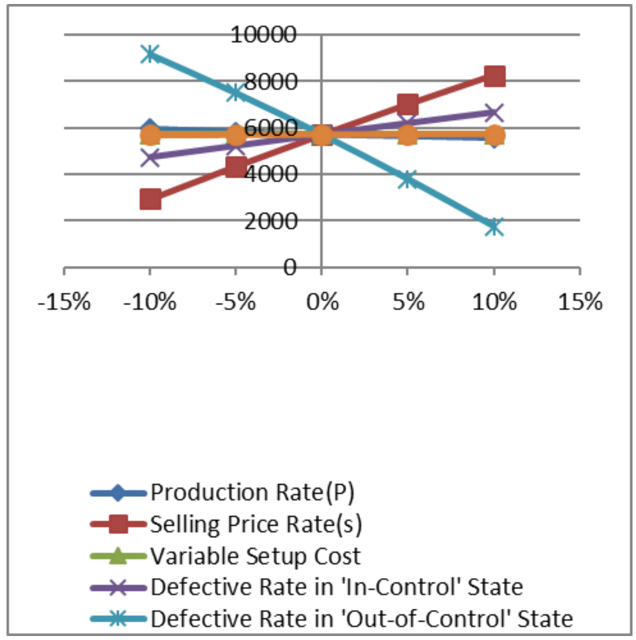

Figure 4: Optimum Profit/unit time with respect to Percentage change of different parameter.

7. Conclusion:

In this study, an imperfect production inventory model with random machine breakdown, selling price discount dependent demand rate are considered. Generally, produces defective item during 'out-of-control' state is more than the produces defective items during "in-control" state due to increases production-run-time because of machinery problems, impatience of labor staff and improper distribution of raw materials. In this point of view, we have considered the defective rate (θ_1) in "in-control" state is less than the defective rate (θ_2) in "out-control" state. The probability distribution of shift time from 'in-control' state to an 'out-of-control' state follows an exponential distribution function. In any production system, machine breakdown occurs during the long run production process because every factors associated with the system are fresh. But, due to continuous running of system these factors gradually losses their perfectness. So, in this direction, we have considered the random machine breakdown. Some portion of product of the produces defective items are reworked at a cost during production run time then to restore its original quality. The objective function of this model is to determine the optimal production run time and expected average profit, solving by numerical techniques. And, the features of key parameters are studied in sensitivity analysis section. The proposed imperfect production system discussed many realistic manufacturing situations, and constitutes a framework for more complex production systems.

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